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Beyond the basket case: A principled approach to the modelling of kagome weave patterns for the fabrication of interlaced lattice structures using straight strips.

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Abstract

This paper explores how computational methods of representation can support and extend kagome handcraft towards the fabrication of interlaced lattice structures in an expanded set of domains, beyond basket making. Through reference to the literature and state of the art, we argue that the instrumentalisation of kagome principles into computational design methods is both timely and relevant; it addresses a growing interest in such structures across design and engineering communities; it also fills a current gap in tools that facilitate design and fabrication investigation across a spectrum of expertise, from the novice to the expert.

The paper describes the underlying topological and geometrical principles of kagome weave, and demonstrates the direct compatibility of these principles to properties of computational triangular meshes and their duals. We employ the known Medial Construction method to generate the weave pattern, edge ‘walking’ methods to consolidate geometry into individual strips, physics based relaxation to achieve a materially informed final geometry and projection to generate fabrication information. Our principle contribution is the combination of these methods to produce a principled workflow that supports design investigation of kagome weave patterns with the constraint of being made using straight strips of material. We evaluate the computational workflow through comparison to physical artefacts constructed ex-ante and ex-post.

1. Introduction

The term “weaving” covers a broad range of textile production methods. Common to all is the principle of material interlacing to generate local systems of friction-based reciprocity. This imbues resulting artifacts with robustness through structural redundancy, resilience through friction-based junctions, efficient use of material and potent aesthetic qualities. These attributes have long been exploited in a diverse range of use arenas, through craft-based tacit knowledge or engineering-based explicit knowledge, to produce lightweight artifacts with emergent properties that offer advantage beyond those of the constituent materials.
1.1 Kagome

*Kagome* represents a particular class of weave which, in many ways, is conceptually closer to braid. Where conventional weave is defined as the interlacing of two distinct sets of yarns (warp and weft) at right angles to each other, braid is defined as the interlacing of three or more distinct sets of yarns (or “weavers”) at oblique angles to each other [1]. In *kagome*, the geometrical archetype arranges these three sets as a regular trihexagonal tiling with a vertex configuration \((3,6)^2\) and \(p6\) symmetry.

![Figure 1: A regular planar sparse *kagome* weave comprising three distinct sets of weavers. The underlying pattern is a trihexagonal tiling.](image)

The physical properties of these lattices are determined by the interplay between combinatorics (valences of vertices and faces, connectivity, and topology), geometry (vertex positions) and material attributes (mechanical and geometric). Tacit understanding of this interplay allows the crafts person to fabricate close approximations of arbitrary design targets.
1.2 Motivation

*Kagome* represents a highly principled method for producing complex curved geometries with a single mesh structure, without the necessity of joinery or the fabrication of nodes. The self-bracing capacity, greater shear resistance (compared to biaxial weave), ability to rigorously control geometry, high redundancy, ability to locally repair and potent aesthetic qualities, make triaxially woven structures an attractive target for investigation across a diverse range of design and craft practices, including architecture. However, without means for visualisation and interrogation, complex design targets can remain challenging for experts to strategise and realise (keeping account of the number of weavers, their crossings and potential self-crossings, calculation of material requirements, assessing discretisation due to material lengths, etc.), and remain out of reach for those without a tacit craft understanding.

*Figure 2:* Triaxially woven structures produced using straight maple strips. Regular (left) and arbitrary (right) geometries are clearly governed by the interplay between introduced topological defects, material stiffness and material geometry.

By intersecting the underlying principles governing the interplay of topology and geometry in triaxial systems with computational representation, a platform for expanded exploration of these systems can be established. This holds relevance to a wide variety of current and emerging domains of application.
In this paper, we present a method for generating weave patterns with the constraint that they be fabricated from straight strips of material. Our motivation for working with straight strips relates to supporting the future exploration of *kagome* applications at scales “beyond the basket”, where efficient use of material becomes a poignant issue. We address key representational challenges including the generation of appropriate topology, or mesh valence, to achieve a design target, together with the relaxation of the mesh to simulate material performance – both of which hold influence over final shape. In addition, we demonstrate the extraction of fabrication instruction and the physical making of computationally developed design targets. We position this work in connection with the literature to: 1) differentiate it from related approaches (specifically related to the use of geodesics); 2) identify the open challenge that our work addresses; 3) cite computational methods that we build upon. Finally, we discuss our contribution, identify its limits and offer trajectories for future work.

**Figure 3:** Two pre-relaxed *kagome* patterns approximating design geometries. The weave topology is directly derived from inherent properties of the design mesh (valence) and the weave pattern directly derived from geometric attributes of the mesh dual (connected edges and their lengths).
2. Topological principles governing kagome geometry

The archetypal *kagome* lattice is a woven version of a tri-hexagonal tiling; the weavers in one direction incline at an angle of 60° to those of the other two directions, and the lattice, consisting of equilateral triangles and regular hexagons, will cover an infinite flat plane (Fig. 1).

2.1 Single curvature

Single curvature of the *kagome* lattice is easily achieved by bending the plane, creating a developable surface. If the axis of curvature exists across the centre points of opposite edges in the unit hexagon, one set of weavers will act as arches perpendicular to this axis. If the axis of curvature exists across opposing vertices of the unit hexagon, one set of weavers will act as beams parallel to this axis. Limits on the radius of curvature are dependent on the mechanical properties of the material.

2.2 Double curvature

Breaking topological symmetry of a regular trihexagonal tiling by the introduction of geometric singularities will induce double curvature [2]. These topological defects, or "lattice disclinations", are the mechanisms that introduce in-plane strains and result in out-of-plane deformation [3]. Positive Gaussian curvature results from the introduction of <6 sided cell. Figure 5 shows physically woven examples in which a single cell has been substituted; firstly with a pentagon, then a quadrilateral and finally a triangle. Of note is the way in which deformation out-of-plane becomes more pronounced as edges are removed from the substituted polygon. Figure 6 shows physically woven examples of negative Gaussian curvature.
curvature resulting from cell substitution with a polygon of side >6; firstly a heptagon, then octagon and finally a nonagon. Here, it is the increase in sides of the substituted polygon that results in a more pronounced curvature. Despite changes in topology through the introduction of disclinations, the vertex valence of the materialised lattice is maintained at $v_4$ throughout, corresponding to the local crossing of two weavers.

Figure 5: Introducing disclination in the regular lattice by substituting a <6 edge count polygon produces positive Gaussian curvature. From top to bottom, each row decreases an edge – pentagon; quadrilateral; triangle.
Figure 6: Introducing disclination in the regular lattice by substituting a >6 edge count polygon produces negative Gaussian curvature. From top to bottom, each row increases an edge – heptagon; octagon; nonagon.

Weaving disclinations provides the means to locally distort the lattice, causing a controlled deformation of the surface out of plane. Strategic combinations of disclinations, informed through tacit knowledge, allow the crafts person to realise specific and diverse design intent (Fig. 7). However, in an inexhaustible space of possible combinations, enlisting computation becomes a relevant tool for exploring, searching and navigating this space.
In this section, we highlight relevant literature restricted to computational representation of weave patterns and related computational methods with a particular focus towards architectural design. We briefly cover methods for establishing and refining mesh topologies, approaches to weave in general, approaches to kagome representation in particular and provide a summary that identifies the open challenge that we address.

3.1 Mesh topology and refinement

With a focus on mesh representations that have relevance to architecture, Schiftner et al. provide a method for refining triangular design meshes such that the incircles of mesh faces form a packing – a CP mesh [4]. This class of mesh is directly related to the kagome pattern, which can be produced by connecting the centres of tangent incircles. As precise CP meshes are rare, an optimisation algorithm is used to refine a mesh towards an approximation of the design target. The mesh is generated by producing an isotropic centroidal Voronoi diagram which is iteratively relaxed using Loyd's algorithm. However, this leads to random placement of singularities which is undesirable if aiming to achieve regular geometries. Use of the mesh operators, edge collapse, edge flipping and edge splitting is a common method for locally refining the topology of mesh as described in Narain et al. [5] and allows approximate locating of required valence in the required position.
3.2 Approaches to weave pattern representation in general

Computational representation of weave patterns in general have been well studied, however, the majority of these relate to biaxial weaving or braiding. In most cases, the representation task is approached using the tiling method described by Mercat [6] in which a predefined tile dictionary defining local weaver geometry and crossings can be applied to a quad mesh. This has been applied in the context of arbitrary manifold design meshes [7], and with specific focus on braided structures [8, 9]. In these cases, the principled approach to representation, which considers interlacing and constraints related to fabrication, provides workflows and tools for realising complex morphologies that are directly producible. However, these tools operate with quad meshes which are less suited to the kagome representation task. In another approaches, modelling proceeds through direct manipulation of explicit geometry [10]. This is not deemed to be a viable approach for the task considered here, considering the opportunity for exploiting the close affinity between the data structures of triangular meshes and kagome pattern principles, and the culture of use surrounding meshes for design expression.

3.3 Approaches to kagome pattern representation in particular

Within architectural design specifically, approaches for defining kagome patterns tend to exhibit shortcomings by either: 1) only considering a topologically regular trihexagonal tiling; 2) exploring geometrical outcomes of fixed and predetermined topologies; 3) abstracting out the weaving principle such that the mechanical properties gained by interlacing are sacrificed, whilst maintaining the topology of the trihexagonal tiling. In the first case (which is often coupled with the third case) complex geometries are achieved by a distortion of the regular grid rather than conforming to the principles for achieving curvature described in the section above [11, 12]. This can present significant challenges for fabrication strategies, junctioning methods and material use. In the second case, relaxation of pre-determined and fixed topologies can result in principled patterns, but impedes fluid design investigation due to a lack of “on-the-fly” topology editing methods.

Kagome patterns have also been explored as a derivative of a general approach to free-form surface segmentation using geodesic pattern
The cited literature describes two approaches – N-patterns from level sets, and the use of a regular trihexagonal web of geodesics – but also identify limits in both cases. Pottman et al. acknowledge that the level set approach produces webs of curves that are as geodesic as possible, but deviations are inevitable in conditions of strong Gaussian variance [13]. Deng et al. point to the fact that true geodesic webs do not exist in general and that adequate surface approximation is not always possible [14].

In contrast to these geodesic methods, which operate from properties of a surface (which in practice is generally approximated by a mesh), our approach operates directly on properties of the mesh and form-finds the final geometry through a relaxation procedure. This models the actions of the local reciprocal systems, which, in practice, we find causes material strips to deviate from true geodesics due to induced torsions often arising in areas of pronounced double curvature. In short, the use of geodesics to derive kagome patterns cannot cover all cases that can be materialised in practice, whereas a principled kagome pattern can always be derived from a manifold triangular mesh [15].

The strong affinity between kagome lattice patterns and computational triangular manifold meshes have been described by Mallos and implemented in the context of a kagome design and fabrication tool [ibid]. However, to our knowledge, this tool does not implement a step that allows the consideration of kagome patterns resulting from straight members – a case that requires relaxation of the kagome geometry with specific relaxation constraints.

### 3.4 Identifying the open challenge

In summary, and in reference to the state-of-art presented here, we can state that whilst there exist a number of methods and algorithms related to the kagome representation task, to the best of the authors knowledge, a holistic computational approach that aids designers by coupling specific fabrication constraints with the principles for “real-time” exploration of arbitrary kagome topologies and geometries, remains an open challenge.
4. Computational approach

Our approach to achieve a principled and generalised method for *kagome* representation, of arbitrary geometries, makes use of various algorithms and methods described in the literature; we declare these below. The contribution of this paper is to draw these together to fulfill the representation task with a focus on fabrication using straight strips of material. The representation task has three stages:

1. topology generation
2. *kagome* pattern generation
3. relaxation to final geometry

4.1 Topology generation

Using the low-polygon modelling method [16], a coarse triangular mesh approximation of the desired geometry is created. In the example shown, the target geometry to model is a *kagome* “socket” condition comprising a regular planar face intersected by a singularly curved tube. The transition exhibits negative Gaussian curvature (Fig. 8). The topology of the low-poly mesh is adjusted to establish the required valence structure. Adjustment is done using conventional mesh refinement operations; edge splitting, edge flipping and edge collapsing [5].

![Figure 8: The target geometry to model is a detail of an existing *kagome* weave with negative Gaussian curvature (left). This is coarsely approximated with a low-polygon mesh (right).](image-url)
Mesh valence of a regular planar tiling is 6, positive Gaussian curvature requires <6 (but >2) valence and negative curvature requires >6 valence. In this case, six valence 7 conditions around the rim of the transition and regular valence 6 conditions to the stem have been introduced. Once the refined valence structure is established, intermediary mesh operations such as relaxation (as in the case shown in Fig. 9) or mesh subdivision can be applied.

4.2 Kagome pattern generation

The mesh dual is obtained and decomposed into a data structure of individual vertices and their three connecting edges. A new vertex is then placed at the centre of each connecting edge and these three new vertices connected with a closed polyline. This operation essentially truncates the original vertex, creating a new facet that represents the triangular element in the Kagome lattice. The operation is equivalent to the medial construction method described by Mallos [15]. At this point, the weave pattern is purely visual and contains no information about weaver continuity; all higher edge faces of the lattice are visually inferred from their surrounding triangles.

The list of truncated face polylines is now converted into a data structure that represents individual weavers. The polylines are exploded into individual linear elements and then “walked” to find connected segments that meet a criteria of minimum angular deviation. Once weavers have been identified, they are locally displaced in an alternating pattern (up/down) along the surface normal vector at crossing points to model interlacing. Once interlaced, each weaver is converted into a triangular mesh approximating the material strip width using the method described by Vestartas et al. [9]. At this stage, meshes may exhibit areas of intersection as can be seen in Figure 11 (right).

4.3 Relaxation to final geometry

The weaver meshes are relaxed using the constraint-based solver Kangaroo2 for Grasshopper. Additional constraints are added to ensure weavers relax into developable geometries approximating straight strips, and to prevent collisions and intersections between weavers – thus preserving the structure of interlacing. Having found the final geometry
Figure 9: The mesh is refined by collapsing, splitting and flipping edges to modify the valence according to the required curvature (left). A preliminary relaxation has then been performed after adding an additional layer of outer triangles in the plane to encapsulate the valence 7 conditions (right).

Figure 10: The mesh dual is obtained (left) and each vertex “truncated” to generate a visual kagome pattern (right). This pattern does not yet describe individual weavers.

Figure 11: The edges of the kagome pattern are “walked” to construct individual weavers (left). Weavers are then displaced normal to the surface to model interlacing, and then meshed according to material geometry (right).
through relaxation, fabrication information can now be extracted (Fig. 12). Weaver lengths are easily determined, and being developable, projected as unrolled strips and marked with crossing points indexed with other weavers or self-intersections. Physical limits on material length can inform weaver discretisation, ensuring sufficient material cross-over for splicing.

5. Two cases

In this section we briefly present two case studies that examine relationships between a computational representation and a physical artefact – one constructed ex-ante and the other ex-post modelling. The first study demonstrates the use of our approach in the context of a simple fabrication exercise. The second study demonstrates the use of our approach in the context of computational design exploration.

Figure 12: The modelled weavers are relaxed to ensure they correspond to straight elements and the final weave geometry is form-found. Fabrication information is then extracted and includes length of strips, strip ID’s and strip crossing ID’s. This information is applied to the weave representation (left) and as material layout (right).
5.1 Case 1: Stadium of revolution

In this first case, we aim to construct a physical weave from computationally generated fabrication information. A stadium of revolution, or “capsule” geometry, is defined as the design target. This geometry comprises a cylinder with single curvature and two hemispherical caps. Drawing upon the principles governing double curvature in kagome lattices, we expect the hemispherical portion to contain pentagonal “defects” to achieve local synclastic curvature. Each pentagon included in the mesh increases the aggregate angular deficiency by $\pi/3$, therefore a triaxial mesh with 6 pentagons will make a hemisphere. The rest of the lattice can be achieved using a regular

![Figure 13: Extraction of fabrication information to produce a woven stadium of revolution.](image)
hexagonal tiling. We follow the modelling steps described in section 4 to
determine how many weavers, their respective lengths, crossings with other
weavers and self-crossings. We see from this analysis that the woven figure
comprises 6 simple rings of length cca. the circumference of the cylinder,
and two longer weavers with multiple self-crossing points. This is verified
with the physically weaving shown in Figure 13 (bottom right).

5.2 Case 2: The distorted helix

In this second case, the *kagome* helix is woven prior to any computational
modelling. Rather than aiming towards verisimilitude of the model, we
demonstrate how the relaxation step can provide exploratory insights
through simulating the interplay of material behaviour and topology. The
helix is modelled and the mesh refined, but in this case disclinations are
randomly placed within the mesh. As the relaxation proceeds and weaver
geometries straighten according to our fabrication constraints, and local
sites of curvature emerge where hexagons have been substituted with
synclastic curvature inducing pentagons, or anticlastic curvature indu-
cing heptagons. In this case, we demonstrate how computation provides
an accessible and fast (compared to physical weaving) exploratory tool
to assist the designer in searching the inexhaustible space of possible
disclination combinations, and potentially discovering novel aesthetic
expressions.

![Figure 14: A physically woven helix with mesh disclinations placed to
realise a regular geometry (left) compared to a simulation where discli-
nations have been randomly located (right). This shows the necessity for
the relaxation step, but also suggests interesting geometric articulations
and “organic” expressions of a corrupted ideal.](image)
6. Towards architectural and structural applications

the instrumentalisation of a principled computational approach to \textit{kagome} pattern generation and representation has broad applicability. Within architecture, hexagonal tiling patterns have been exploited to stunning spatial effect by Shigeru Ban in projects such as the Pompidou Metz and Nine Bridges golf club. However, in these cases, double curvature is achieved through a distortion of the regular hexagonal tiling. The resulting geometry is realised through complex shaping of stiff curved laminated members. In such a context, the application of \textit{kagome} topology principles for achieving complex geometry could offer a more rational approach to geometry with the implication of greater efficiency in fabrication.

In the context of elastically bent structures, the attributes of mechanical performance arising from interlaced material and efficient spanning of space with straight strips of material have been demonstrated in the CODA Jukbuin Pavilion. In this case, the weave principle of material interlacing is maintained but double curvature is achieved through material bending behaviour rather than steered by topology – the design topology is a regular hexagonal tiling. This results in global curvature effects but denies the possibility of highly localised double curvature. Nevertheless, this work is of particular interest as it demonstrates the transfer of interlacing principles at architectural scale.

In framing a direction for future work, our emerging hypothesis is that architectural scale structures can be realised with full adherence to \textit{kagome} weaving principles, including material interlacing. This hypothesis is supported by a comparative analysis of two hypothetical gridshells which shows that a \textit{kagome} gridshell outperforms a quadrilateral gridshell for a very similar construction cost [17].

Our outlook is towards the use of elastically bent members rather than stiff curved laminated members. However, as we discuss above, we see \textit{kagome} principles being applicable in both contexts - in the former, towards bending-active structures that adhere more closely to their basket antecedents; in the latter, towards rationalised approaches to geometry and fabrication. In the context of elastically bent structures, principle challenges revolve around structural capacity. Yet despite this challenge, the opportunities for material efficiency, a rationalised approach to free-form geometry and efficient fabrication minimising the use of connectors make this a compelling territory for further study.
6.1 Limits and future work

Where the work presented in this paper has limited itself to exploring the task of *kagome* representation and simulation with consideration to fabrication constraints, analysis of structural performance marks a necessary next step – especially if seeking to explore architectural applications. Preliminary investigations of model transfer to the structural analysis platform Autodesk Robot indicate that representational outcomes generated by the approach described are poised to be taken forward into this domain of analysis. In addition, the ability to computationally represent arbitrary *kagome* geometries and interrogate these from a fabrication perspective, presents the compelling opportunity of investigating robotic production.

7. Conclusion

This paper has presented a principled computational approach to the task of *kagome* representation in arbitrary triangular meshes. Following the literature, we have demonstrated the strong affinity between the principles governing *kagome* patterns and intrinsic topological features of computational meshes and geometric features of their duals. We have shown how design meshes can be manipulated to adjust the baseline valence 6 structure that governs planar *kagome* tiling, upwards and downwards to create sites of local double curvature. We have also shown how the *kagome* pattern itself is derived from the mesh dual by vertex truncation to the mid-points of connected edges – following the medial construction method.

We have extended the state-of-the-art by intersecting this method with physics based relaxation to allow simulation of the interplay between topology and notional mechanical properties of weaver material, thereby constraining results within the bounds of fabrication criteria – specifically that patterns can be made from straight strips of material. This constraint is seen to be a benefit for enticing transferability and use within domains where material saving can be a key issue, such as architecture.

Finally, the approach presented here contributes a method that can be computationally leveraged to explore and search the inexhaustible domain of possible *kagome* patterns, and opening the possibility of this search to be conducted by both the novice and the expert.
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