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Interactive Strain Mapping for Non-Rigid Folding Origami Structure

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Abstract

The focus of the current research is at developing a tool, which can provide more readily available information about the dynamic deformation behavior of the structure based on instantaneous average strain of each facet in the earlier design process. The aim is not to provide the most accurate strain as it requires greater resources, but to inform the relative strain levels in the facets at the instant time frames during the folding/unfolding process.

The current investigation started by considering origami structures with simple plane elements, and assumed that the edges are in full contact along the lengths. At the current stage of development, only in-plane deformations are considered.

The investigation first looked at the generation of origami pattern based on the user defined geometrical surface. For the given pattern, the origami structure is forced into an initial 'fully unfolded state'. The geometrical profiles of each origami facets at the 'fully unfolded state' are set as the '0 state profiles'.

The geometrical configuration of the origami structure can be then interactively changed by moving a reference points or frames, or by 'pulling' the structure into specifically defined geometrical configurations. During the changes in the geometrical configuration of the origami element, any deformations of the individual facets are computed against the '0 state profiles' for strain calculation.

In the last stage, the tool puts the simulated structure through an optimization process with minimum strain and number of facets as competitive fitness criteria, with the aim to provide a feedback to the designer on how the folding pattern can be improved.

The current paper includes the case of a stent fold, and the tool is developed in Grasshopper and Kangaroo platform, which are graphical algorithm plug-ins for Rhino 3D.

Keywords: Origami, Parametric, Simulation, Grasshopper, Kangaroo

1. Introduction

The current paper is exploring the application of graphical algorithm plug-ins for early stage design tool for origami structures. The investigation explores the use of Grasshopper® and Kangaroo, which are the graphical algorithm tools developed for 3D CAD program name Rhinoceros®.

Rhinoceros® has substantially extended its competency with the development of Grasshopper®, which is now extensively used by global designers from various fields, and has shown the tool's great capability in parametric surface modelling. Kangaroo was later developed by Daniel Piker as a physics engine plug-in for Rhinoceros® and Grasshopper® which allows designer to simulate selected aspects of real-world behavior. Kangaroo is based on particle-spring system that depicts structural systems using network of particles, with which certain spring stiffness defined between. The immediate advantage of the integrated physics engine in computer-aided design (CAD) tool is the absent process of exporting/importing models between CAD and structural analysis software. Especially when there are increased number of models to examine effects of different parametric changes, the use of CAD tool with integrated physics engine can increase the work efficiency. Surely, such tool cannot be considered as an alternative to structural analysis software due to the limited functions, but more as early stage design-aid solutions. Therefore the actual discussion is at appropriate development of the tool with respect to the following key objectives:

- To provide sufficient insights to structural behavior towards the initial confirmation of geometrical design at early design stage.

- To provide initial understanding of structural behavior in respect of strain output with known approximate levels of discrepancy with the outputs of structural analysis software.

- To provide alternative design solutions with respect to user-defined variables.

Origami, the art of paper folding has inspired architects and engineers to design both static and kinetic (ones that can fold and unfold) forms, ranging from roof structures with increased stiffness to inflatable booms for deployable space structures. Many research has been done in the past in the area of form-finding and dynamic analysis of origami structures. For kinetic origami structures that are typically constructed with panels and one-DOF hinges, the possible number of the foldable forms are affected by the actual rigidity of the panels. In other words, for completely rigid panels there exits only strict number of crease patterns that make the origami structures foldable, and any involvement of excessive forces to fold/unfold non-foldable will only lead to either breakages of the hinges or local failure of the rigid panels. However, with increasing flexibility of the panels (hence they can accommodate certain degree of deformation during folding/unfolding process), more foldable forms become possible which were not with rigid panels.

2. Generation of Origami Pattern using Grasshopper

The choice of grasshopper for the parametric design tool was in respect of the flexibility and interactive design exploration. The written algorithm in grasshopper provides the flexibility of generating different geometries by easily changing specific geometrical parameters. As the specific parameters can be defined using clusters such as ‘slider’, the manual typing process by the users is further reduced. Thus, large number of geometrical variations can be produced without the need to modify the existing drawing.

Furthermore, with additional algorithmic clusters in Grasshopper the initially simple geometrical parametrisation can be integrated with advanced computations including dynamic geometrical simulations or geometrical optimisations with user defined criteria. Combining geometry generation, performance based simulations and optimizations computation into a single platform promotes greater work efficiency, and allows the designer to be in greater control of the entire process.

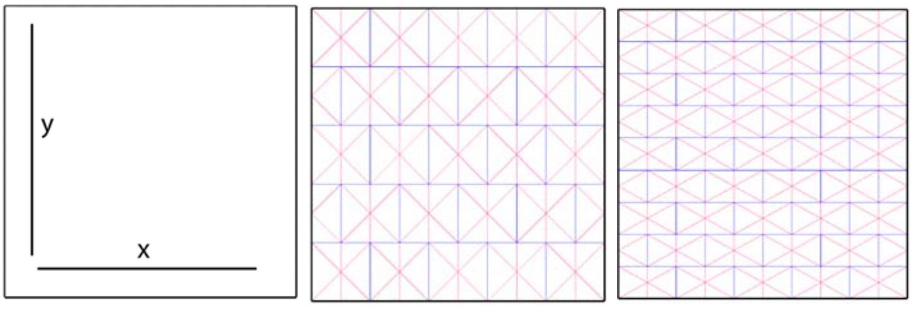

Figure 1. Generation of different Stent origami patterns: Blue and Pink lines represent hills and valleys: Different grid densities of the pattern can be generated in fraction of second by changing the number of elements in x and y axis

2.1 Particle-Spring System in Kangaroo

Kangaroo solver uses dynamic relaxation and seeks for equilibrium at particles while satisfying individually set 'goals' (e.g. stiffness between particles or boundary constraints) in the system of particles and springs. The goal definitions can be developed in reflection of material and structural principles, and thus, more realistic goal definitions will lead to more accurate simulation results, which may more closely represent its structural behaviour in real environment.

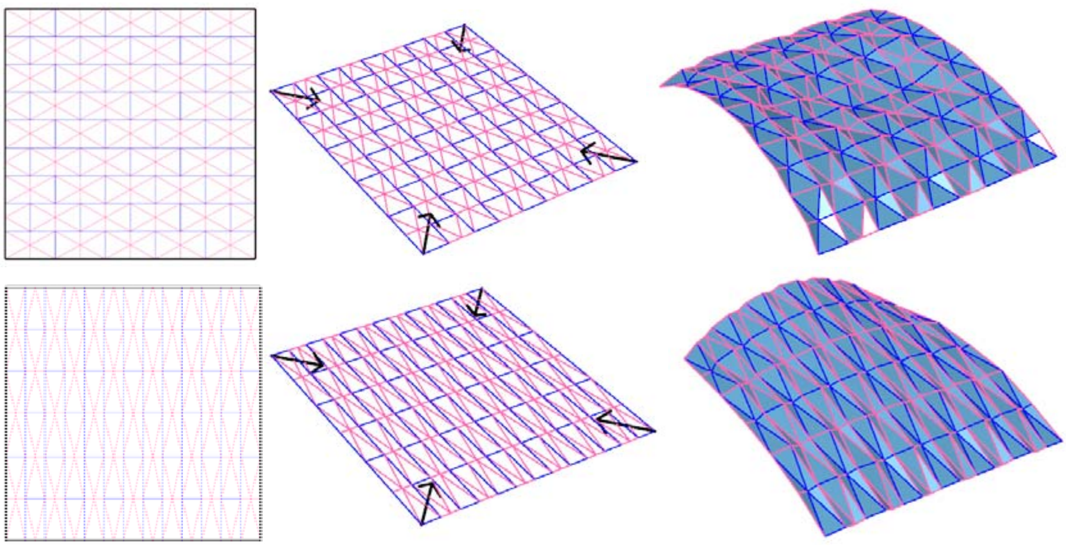

Figure 2. Simulating the fold of different grid densities with the same forces applied at the corners.

For folding simulation of origami structures, there are two important goal definitions among others: 1) rotational stiffness at common edge (hinge) of two adjacent facets and 2) material stiffness of each facet. Often the hinge stiffness is set to zero in simulations, unless more accurate reference value is known based on the actual friction. However, another important purpose of the hinge stiffness is to make sure that the facets fold in the intended direction, and form correct either mountain or valley fold.

2.2 Framework of Origami Facets

For analysis of origami structures in account for the dynamic deformation of facets, the use of discretized FEM models can require greater computing power. Especially for earlier design stage it is recommended to use simplified calculation models. For example, in 1943 A. Hrennikoff described the Framework Method, which replace continuous solid plate with a framework of elastic bars that are configured in a certain pattern to reproduce the overall stiffness behavior of the solid plate [1]. Hrennikoff suggested different methods to define the areas for both quadrangular and triangular elements; though for triangular elements, the method was specifically for equilateral triangles with poisson's ratio of 1/3.

A number of recent works discussed modelling quadrangular plate using angular spring stiffness [2] or rotational hinges [3] for simulating the flexure behavior of panels. In the work by Filipov et al, a panel model with five node and eight bars (N5B8) were introduced to overcome an-isotropic in-plane deformation of N4B5 model by Schenk and Guest [4] and restricted bending displacements of N4B6 model. The evolution of panel models from N4B5 to N5B8 shows the apparent pattern of increasing level of discretization with increase number of bars and the node, and one can immediately notice the indifferent model characteristics of which can exit in the particle-spring system. In the bar and hinge systems, the bar stiffness, k is defined by EA/L, where L is the bar length and A is the bar area. For modelling quadrangle with internal diagonals, the bar areas of horizontal bars, A_x, vertical bars, A_y and diagonal bars A_d are defined as [3]:

$$A_x = t\frac{H^2 - vW^2}{2H(1-v^2)}, A_y = t\frac{W^2 - vH^2}{2W(1-v^2)}, A_D = t\frac{v(H^2+W^2)^{\frac{3}{2}}}{2HW(1-v^2)} \tag{1}$$

Where, v is poisson's ratio. The spring stiffness can be calculated for quadrangle elements using the same area defined above. In case of triangular elements, the above equation for diagonal members may be used with slight modification as shown below:

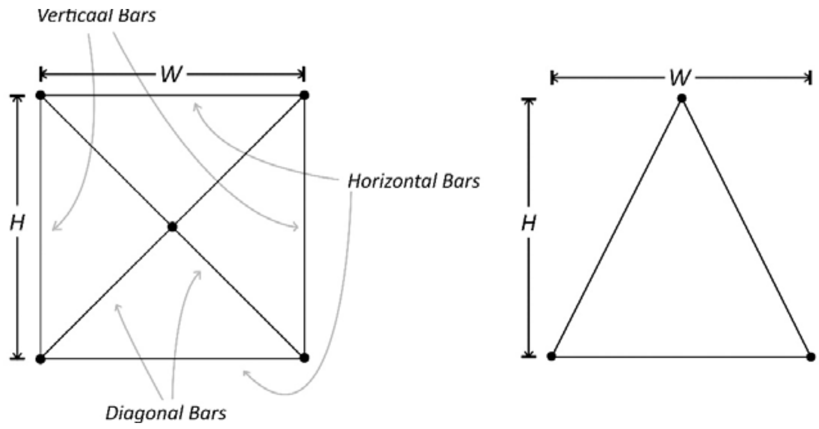

Figure 3. Rectangular Element N5B8 (Right) and Triangular Element (Left)

$$A_t = t\,\frac{v(H^2+W^2)^{\frac{3}{2}}}{\sqrt{2}HW(1-v^2)} \tag{2}$$

The equation results in accurate prediction for tension load when compared with FEM results (Figure 4), but underestimate the displacement (overestimate the stiffness) for shear load by approximately 50% (Figure 5), which is also the case for the N5B8 quadrangle element as stated by Filipov et al [3]. The same results are obtained in use of Hrennikoff's area calculation method. As the current investigation explores origami structures with triangular facets with the above area calculation method is used.

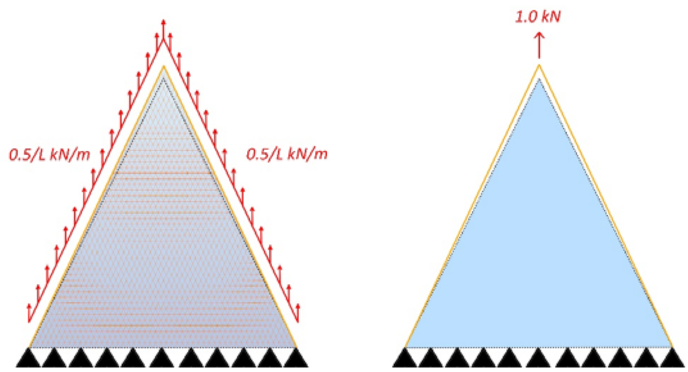

Figure 4. Triangular Element with Tension Load: FEM (Right) and Spring-Particle Model (Left). The yellow lines depicted deformed shapes in 10,000 structure scale.

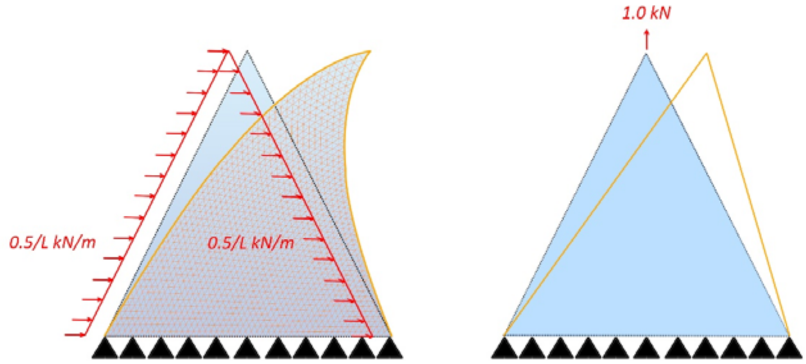

Figure 5. Triangular Element with Horizontal Load: FEM (Right) and Spring-Particle Model (Left). The yellow lines depicted deformed shapes in 10,000 structure scale.

3. Strain of Facets

Spring-particle system represents structures with simplified framework of particles connected by springs with the defined stiffness. Hence, for triangular elements, the resultant geometry of connecting three particles (nodes) are always flat, and the strains by out-of-plane deformations cannot be modelled without further discretization of the frameworks. This discretization has to be accompanied by updated area definitions as well as the bending (out-of-plane) stiffness between linearly connected springs. At present, the bending stiffness can be modelled using 'Bend' or 'Polylinebend' clusters in Kangaroo and with future development of adequate spring area calculation method and equivalent bending stiffness, it is expected to analyse thin plate elements based on out-of-plane deformations.

Despite the limited capacity to assess only the in-plane strains, it can be still argued that in-plane deformation of facets becomes the relevant criteria to assess as triangular elements remain planar during folding due to high-bending resistance of folded edges of the elements. The strains (ε_x, ε_y, ε_{xy}) are calculated based on simple strain-displacement relationship as described below:

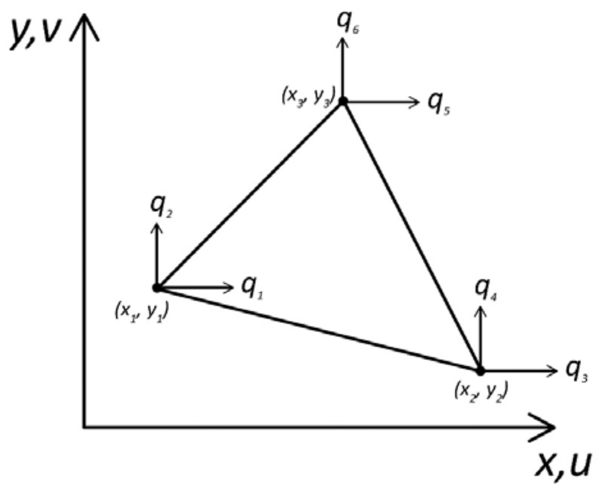

Figure 6. Notation of Triangular Element

$$\varepsilon = \begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{pmatrix} = \text{Bq, where, B} = \frac{1}{\det J}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & y_{32} & 0 & y_{13} & 0 & y_{21} \\ y_{32} & y_{23} & y_{13} & y_{31} & y_{21} & y_{12} \end{bmatrix}, \text{ q=}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} \tag{3}$$

It can be generally agreed that as the model generation of linear strain triangle with three more nodes is not greatly more difficult, and would result in better approximation. However, in-plane stiffness of triangular element will result in small strain gradient, and as the tool aims for more rapid preliminary design with a known level of discrepancy, the use of CST can be acceptable.

4. Evolutionary Optimization

There can be a vast number of different folding patterns for a given surface geometry. However for a given type of folding pattern (for example, Miura Pattern or Yoshimura Pattern), it is understood that an optimal geometrical configuration of the facets can exist with minimum strain in the deformed facets with respect to external loads. At present there exists a very inspiring tool, called Freeform Origami, which was developed by Tomohiro Tachi. The tool allows the designer to interactively edit the overall geometrical form of origami by pulling vertices in rather an arbitrary manner, and inform the final crease pattern. In acquaintance of the tool, the current investigation explores a possible tool, which can inform the user the final design of origami structure based on the result of evolution optimization towards minimum strain in the facets with respect to the specific load conditions. Similar approach was used by D. Nha Chu to minimize the weight of structures while satisfying stiffness requirements. [5]

The tool is designed to run two phase optimization. The first phase optimization is carried to fit the criteria of minimum strain in respect of the minimum number facet elements. It is to note that the level of minimum strain in the facets can be lowered with increased number of total facet elements in the structure. This could lead to the generation of hundreds of facets, which may not be realistic with respect to the actual fabrication of the overall structure. Thus, additional criteria is required to limit the maximum number of facets that are explored during the optimization process. This criteria can be in respect of user defined conditions, for example, maximum facet size for transportation or maximum number of hinge connections for labor efficiency. For the current study, an arbitrary number of 20x20 was chosen for the total number of facets. Please note that the choice of number of facets as a fitness criteria was for the study purpose only. In the grasshopper platform, any further fitness criteria can be added or existing criteria can be modified at any point of design development; which is again a great strength of the platform.

In order to allow the optimization to explore the a wider spectrum of possibilities, the parameters of the evolution had to be set up accordingly to the goal – low *Elitism* (percentage of new solutions that are bred out of the Elite instead of the entire pool, when set high, more local optimization is performed); high *Mutation Probability* (the probability of each parameter (gene) to become mutated at the mutation rate); high *Mutation Rate (*low mutation rate means little changes to the parameters' values, a high rate means big changes); mid *Crossover Rate* (the probability of two subsequently generated solutions to exchange parameter values).(Figure 7)

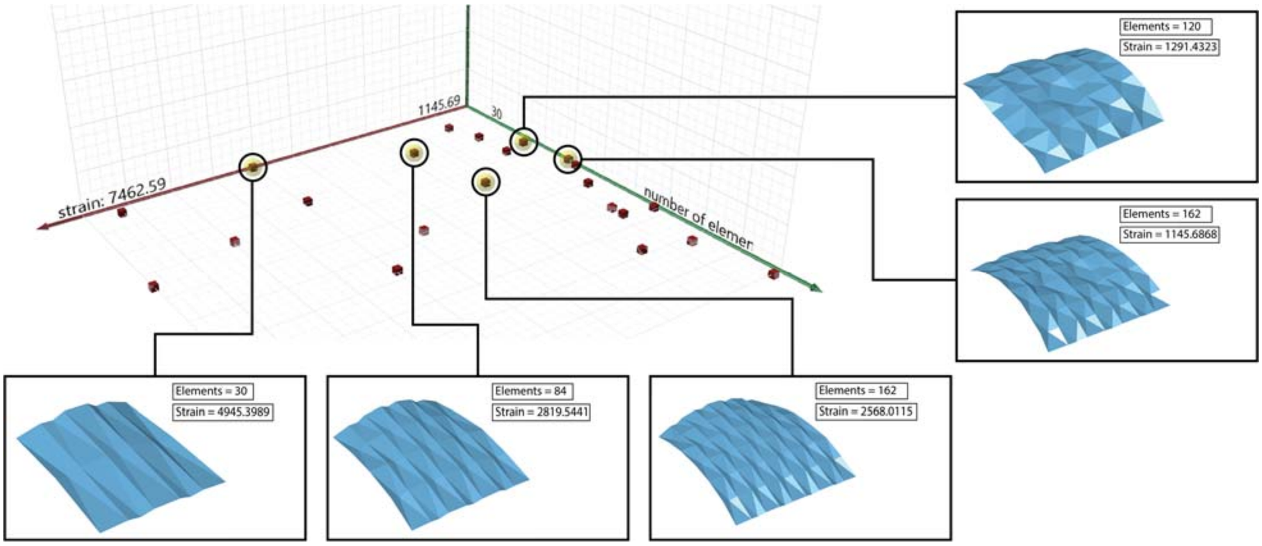

Figure 7. Map of the solution, rated by their number of elements and the strain.

The follow-up optimization introduced local geometrical changes of the elements, based on distance to attractor points. Here the algorithm defines points on the surface of the flat origami boundary using local UV coordinates. Then based on the distance of each of the element's vertices, to the closest point, a deformation is introduced, as the actual amount is chosen by the algorithm. Then the geometry is folded and the strain is analyzed. The vertices are either moved toward the closes point, or away from it, while keeping them inside the chosen boundary. (Figure 8)

Since there are dozens of ways to pick the points on the surface as well as vast number ways to move the points of the origami, a different optimization strategy was needed. Generating all the possible solutions here was not an efficient way. Also the intuition of how much and where do we need to deform the pattern is lacking. For this reason the optimization had to *find* the most fitted solution. The parameters of the evolutionary solver were: high elitism, mid mutation probability, mid-low mutation rate and high crossover. This case also called for generating more than one generation, which enhanced the search for the best fitted. A total of 245 solutions in 7 generations were examined.

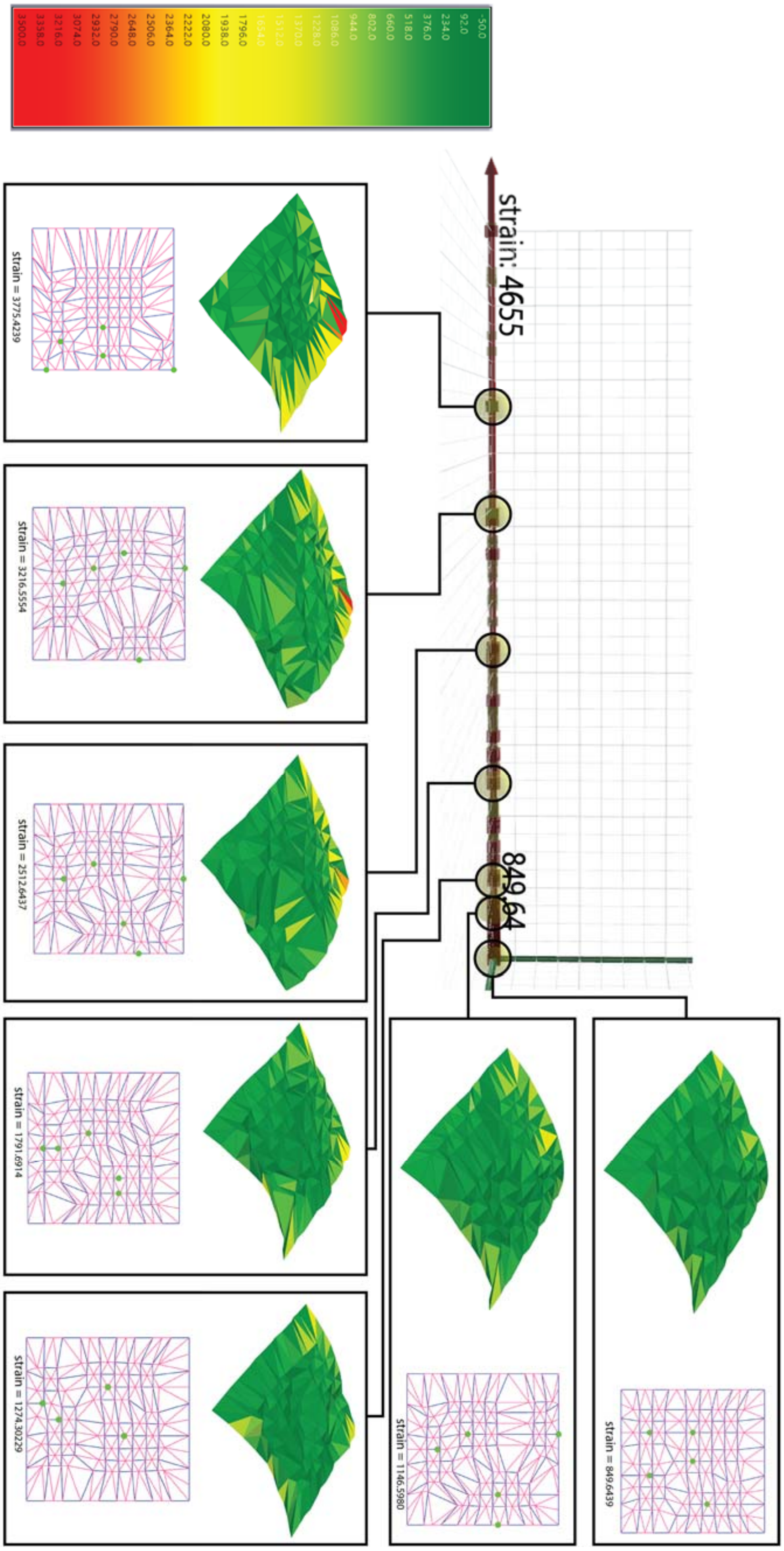

Figure 8. Follow-up optimization with the elements mapped by strain .The deformed unfolded pattern is also shown.

5. Conclusion & Discussion

The current paper presents a work in the development of computational tool for preliminary design stage of origami structures. It is the intention for the tool to provide the approximated, but relevant information on the overall deformation behavior of origami structures, and more optimized design solutions based on reduced strain level and other user-defined parameters.

During the development of the current tool, it could not be clearly defined the desired level of approximation, to which the structural simulation is developed. For accuracy, there are more sophisticated commercial tools available, though it may come with greater investment of resources. The accuracy of the current tool can be improved continuously by including additional algorithms, yet the results would be proportional increase in the computation time, and this is also not desired as the initial aim was at rapid assembly of approximated solutions for preliminary decision making process.

The eventual conclusion was at the future development of simulation, which would produce results at consistent discrepancy with advanced FEM software and/or physical test results.

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