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Published in: Proceedings of the International Association for Shell and Spatial Structures (IASS)

*Publication date:* 2015

Document Version: Publisher's PDF, also known as Version of record

Link to publication

Citation for pulished version (APA):

Holden Deleuran, A., Schmeck, M., Charles Quinn, G., Gengnagel, C., Tamke, M., & Ramsgaard Thomsen, M. (2015). The Tower: Modelling, Analysis and Construction of Bending Active Tensile Membrane Hybrid Structures. In *Proceedings of the International Association for Shell and Spatial Structures (IASS): Future Visions* 

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Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2015, Amsterdam Future Visions 17 - 20 August 2015, Amsterdam, The Netherlands

# The Tower: Modelling, Analysis and Construction of Bending Active Tensile Membrane Hybrid Structures

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## Abstract

The project is the result of an interdisciplinary research collaboration between CITA, KET and Fibrenamics exploring the design of integrated hybrid structures employing bending active elements and tensile membranes with bespoke material properties and detailing. Hybrid structures are defined here as combining two or more structural concepts and materials together to create a stronger whole. The paper presents the methods used and developed for design, simulation, evaluation and production, as well as the challenges and obstacles to overcome to build a complex hybrid tower structure in an outside context.

**Keywords**: hybrid structures, computational design modelling, form finding, bending active, bespoke knit, finite element analysis

# 1. Introduction and Design Principle

The Tower is a hybrid structural system constructed from stacking overlapping glass fibre reinforced plastic rods embedded in a bespoke knitted membrane made from high tenacity yarn. Knitting enables the inclusion of detailing for joining and tensioning the system into the membrane itself. The tower is a form active structural system exploring the potential of combining bending and tensile members for architectural design. Compared to static and homogenous systems this involves an increased level of complexity in terms of modelling, analysis, fabrication and construction. Our research aims to examine how architects and engineers may collaboratively engage with this challenge. We approach this under the working hypothesis that developing new computational design models for implementing feedback between different scales of engagement can lead to better, more creative and resilient building practices. The project explores how design intent and feedback can be passed between different scales of engagement and modelling. Here the project defines three central design challenges: At the **macro** scale of the structure the architectural typology of a hybrid tower presents challenges outside of common applications of form finding, such as shells and membranes. At the

**meso** scale of the elements the project explores the potential of bending active tensile membrane structures as a strategy for increasing resilience through actively deforming structures. At the **micro** scale of the material the project introduces bespoke knit as the tensile membrane. The project investigates if this type of construction is sufficiently strong and if it is able to create a continuous force flow between discrete elements in the structure, circumventing the obstructive effect of stiff connections between active bending members [13]. The membrane between the rods of each layer is radially pulled to the tower central axis. This results in a spoke wheel effect which provides horizontal stiffness and braces the rods which carry vertical loads. The rods interface the membrane in pockets and tubes integrated in the knit. The tower is a soft structure - flexible and bendable - capable of responding to impact and its changes in its environment. This inherent flexibility is considered as a property of potential resilience. That is, the ability to recover from or adjust to change of external stimuli. Focusing on the strain of live loads from wind, the soft structure stores energy when it is deformed elastically and releases that energy upon recovery.



Figure 1: The Tower in the Courtyard of the Danish Design Museum. The interior of the Tower is characterised by the tensioning system and the resulting cone-like membranes. Photo: Anders Ingvartsen.:

# 2. Computational Design Modelling

## 2.1. Development Challenges

The project contributes to two modelling challenges: 1) interfacing multiple heterogeneous computational design models in pipelines characterised by cyclic dataflow, 2) resolving design cases which are inherently complex and require phenomena to be modelled as the product of local interacting behaviours. The first challenge probes how we interface generative and analytical design models in integrated modelling pipelines with the goal of improving the design space search delineated by the models. The second challenge explores how to model the hybrid behaviour of bending active tensile membrane structures and its practical implementation in an interactive form-

finding model. In terms of developing a robust, fast and flexible design modelling pipeline several inquiries were examined:

- How to form find hybrid bending active tensile membrane structures?
- Which constraint solvers meet our requirements and how do we implement them?
- How do we establish a flexible logic for generating the tower topology?
- How do we define meaningful modes of analysing form found geometry?
- How do we develop relevant and bespoke descriptions for fabrication?

#### 2.2. Form Finding Hybrid Bending Active Tensile Membrane Structures

During the form finding of membranes, the membrane is set to a fraction of its real stiffness in order to facilitate large deformations. The resultant shape represents the pure flow of forces. In a hybrid system a different type of element is added. Unlike the membrane, the rods maintain their bending stiffness during form finding. The higher the curvature and the diameter of the rods, the higher the stresses in the material. The material of the rods needs to have a high strength and a low Young's modulus to be able to perform accordingly. It is important to create a curvature high enough to tension the membrane without risking breaking the rod under excessive bending. Under influence of external loading, deflections are quite big and can exceed the bearing capacity of the structure's elements. That is why structural analysis is essential to guarantee the performance of the tower. In case of exceeding the bearing capacity, the design of the structure has to be revised.

The FE environment (Sofistik) is suited to the real-time form finding of complex structural systems with large deformations; however it does provide a precise mathematical definition of the global stiffness matrix. Large deformations must be simulated using an incremental process. To bend beamlike elements into shape, the *elastic cable approach*, developed by Julian Lienhard [11], was used. Single element elastic cables are connected to the initially un-deformed rod then a pre-tension force is applied to the cable causing it to shorten in length and subsequently pull the cable ends towards one another. After the rod is pulled to its defined curvature, the membrane is then fixed to the deformed rod, and the system is released. The rod then tries to straighten again, subsequently tensioning the membrane while the membrane restrains the rod and both find their equilibrium. With a complex system like the tower, this method quickly hits its limits. Therefore, a two-stage process was used. In a first step form-finding is carried out in the Grasshopper environment using a tool based on a mass spring system (MSS). The high stability and speed of the process allows an experimental approach which gives almost real-time feedback. The disadvantage is that the axial and bending stiffness definitions in Kangaroo are dependent on assorted mathematical approximations whose numerical precision is still undergoing validation. After completing form finding, the geometry can be baked and exported to the FE-environment (Sofistik). Here the lines and surfaces are converted into structural beam and membrane elements and materials, cross-sections and support conditions are defined. In a second step the stresses from the FE simulation are superimposed with stresses from the form-finding. The stress within the bent rod depends on the curvature, the diameter and the Young's modulus of the used material and is expressed by the following simple equation. Adding the resultant stress gives a good impression of the overall stress level including bending and external impact

$$\sigma = \frac{\mathbf{E} \cdot \mathbf{t}}{2 \mathbf{r}} \qquad [\frac{\frac{\mathbf{N}}{\mathrm{mm}^2} \cdot \mathrm{mm}}{\mathrm{mm}}]$$



Figure 2: The elastic cable approach for bending beam elements generating highly accurate residual stresses.

#### 2.3. Modelling Precedence and Implementation

The Tower extends research by CITA/KET exploring computational modelling of actively deforming structures (Deleuran et al. [9], Quinn et al. [15], Alpermann & Gengnagel [4] and is related to work on hybrid structures by Ahlquist & Menges [2][3], Lienhard [11] and Mele et al. [12]. We built upon this by similarly implementing a particle based constraint solver operating on discrete piecewise linear geometries for modelling the behaviour of bending members and tensile membranes in one unified and interactive system. This is integrated within a larger modelling pipeline in the Grasshopper environment of the Rhino 3D CAD package. There are several solvers available for Grasshopper including Kangaroo [15] and ShapeOp [8]. These model bending behaviour accurately but require uniform discretisation. This adversely affects the freedom with which to construct the input geometry and has the consequence that one must dimension the members relative to each other prior to form finding. We were fortunate to get involved in the early stage testing of Kangaroo2 (codenamed Joey). It improves upon existing solvers on several levels instrumental to the project: 1) The API is designed for being implemented through scripting, allowing us to develop a bespoke, minimal and optimised pipeline. 2) Constraint weights can be set arbitrarily high and still remain stable, enabling the simulation of stiff materials with fast convergence. 3) The bending constraint implements the resolution independent Adriaenssens and Barnes model [1], enabling the modelling of non-uniformly discretised bending members.



Figure 3: Early prototype demonstrating basic modelling, analysis and fabrication results

2.4.

#### 2.5. Computational Design and Form Finding Modelling

The computational design modelling pipeline is divided into five parts integrated in one multi-stage Grasshopper definition. The central algorithms and functionality are implemented as GHPython components which implement the RhinoCommon and JoeyPhysics libraries:



Figure 4: The computational design modelling pipeline on the Rhino/Grasshopper side.

#### 2.5.1. Generate Tower Topology and Member Geometries

The design principle of stacking overlapping bending members around a central vertical axis is used as the geometrical principle for generating tower topologies. The model input variables for this process is a list of values which sets the number of sides, the side length and the number of floors. The model outputs equilateral polylines representing the un-discretised bending members and membrane cells. A list of membrane cells forms a membrane patch spiralling around the tower. These polylines are further processed to generate the geometries which form the input for the form finding process. This process has three models which generate the bending member polylines, the membrane patch meshes and the tension member lines. The bending members are discretised and extended to overlap with bending members from one floor to the next. A second layer of bending members is added to the ground floor and the anchoring points of these layers are moved apart. The membrane cells are used to generate and subdivide meshes representing the knit patches. The tension system is generated by cross-referencing the centroids of the membrane cells with the membrane patch meshes.

## 2.5.2. Form Finding and Bending Member Dimensioning

The form finding and dimensioning process has three stages: generating, exercising, and refining constraints. A bending member is represented by a spring constraint for each edge maintaining its length, and, a bending constraint for each vertex along the polyline and its neighbours which tries to keep the angle formed by the three points at 180 degrees. A membrane patch is represented by a spring for each edge which minimises its length. Naked edges are given a weight multiplier, enabling the effect of tension wires along the membrane perimeter. The internal tension system is represented as springs. Constraints are fed to a component which allows the designer to interactively manipulate

constraint values and geometries. The component uses the Joey physical system to iteratively solve the constraints in a feedback loop and converge to a state of equilibrium. The designer manipulates two lists of constraint values, on a per floor basis, which dynamically dimension the bending member lengths and the distances along the bending members where they intersect their neighbours. This enables the designer to control the macro shape of the tower using exact member dimensioning. The value lists are implemented as scripted "gene pools" which automatically adapt to changes in the tower topology. The process outputs the form found geometries and solver statistics. The bending member results were verified using 3D scanning of physical prototypes.



Figure 5: Three instances of tower topologies tagged by genotype. The local polyline members of the system are highlighted in the fourth image, followed by the three structural member types and their geometric representations.



Figure 6: Five steps in the iterative and interactive form finding and dimensioning process.

#### 2.5.3. Analysing Form Found Geometries

The pipeline has two analysis models which guide the designer towards design instances which may perform better in relation to structural performance. A desired geometric property in membrane design

is high double curvature as this stabilises the membrane. This property is analysed by a component which returns the local curvature of each vertex and visualises it in the viewport. For the bending members a key geometric property with structural implications is the local bending radii. This value can be mapped to the bending stress, utilisation and reserve of the member in isolation of other load cases. The local radius is defined here as the radius of the circle constructed through a polyline vertex and its two neighbours. This property is visualised in the viewport as coloured/scaled vectors which also provide a visual representation of the bending orientation.



Figure 7: Comparative bending radii analysis of differently dimensioned towers. Note the relationship between macro shape and bending radii.



Figure 8: Developing a membrane in the XY-plane. The values indicate differences between the form found and the in-plane meshes (MAD = Mesh Area Difference, TELD = Total Edge Length Difference).

#### 2.5.4. Developing Membrane Patches in the Plane

Getting the membranes into the plane for fabrication is challenging as they are double curved and may not be discretised into panels as with conventional membrane design. Instead we developed a constraint-based approach implementing Joey. A form found membrane mesh is input to a process which also inputs a planar topological identical mesh with equilateral edges. A constraint is generated for each vertex of the planar mesh restraining it to the XY plane. Each edge of the planar mesh is constrained to the length as its corresponding edge in the form found mesh. The constraints are solved, outputting a planar mesh which is nearly metrically identically to the form found mesh based on area difference and total edge length difference. The bounding box of the output mesh is minimised using the Galapagos evolutionary solver to ensure that it will fit the knitting machine.

## 3. Structural and FE Modelling

#### 3.1. Superpositioning stresses

The total stress in the system to be evaluated is the addition of the residual stresses from form-finding [FF], plus the residual stresses from membrane pre-stress [P] and finally stresses from the static load cases of dead load [DL] and wind [W]. As described in sections 2.2 and 2.4.3 the stresses from the form finding [FF] have not been calculated in the global model but are taken from a bending radius analysis. Therefore the results of the FE-model do not include residual stresses from the form finding.

$$Total Stress = [FF] + [P] + [DL] + [W]$$

The larger the diameter, the smaller the stress reserve for the bending radii in our structure. This means that a delicate balance is needed to be found between section diameter and internal stress reserves.

#### 3.2. Material Properties

For the FE model, the material properties for GFRP rods were taken from the datasheets given by the manufacturer. As the properties of the knitted material were to be designed during the project, properties of a standard PVC Type I membrane with a thickness of 0.8mm were taken as a point of departure. Tests of the biaxial behaviour of the fabric knitted from a high strength polyester yarn at the *University* Duisburg-Essen, Laboratory for Lightweight Structures showed however significant differences between the designed knitted material and our first assumptions on which we build our structural concept.

	Young's Modulus	[N/mm <sup>2</sup> ]	Poisson's Ratio[-]	
PVC Type 1 Membrane	500	50 times	0,2	3-4 times
Tested Fabric	6,25-13,0	lower	0,66-0,83	higher

Once finally available, definitive test results of the knitted fabric indicated that its stiffness was approximately 50 times lower than that of a standard PVC1 membrane. While a FE simulation with these lower stiffness properties was attempted, it became quickly apparent that the global stiffness was far too low and subsequently the simulation failed. While the yarn of the knit is sufficiently strong, the

difference in behaviour between knit and woven, laminated membrane can be explained through the structure of the material: in a woven membrane: multiple parallel fibres are laid out rectilinearly in a more or less straight fashion and curve only slightly around each other. Membrane material therefore has almost no geometric stretch in the two fibre directions under tension. In knitted fabrics one continuous yarn runs in sinusoidal loops. Under tension, these sinusoidal yarns are pulled straight, which leads to a large amount of stretch. Only after "locking" in this linear configuration the material starts to bear loads in a linear-elastic manner.

#### 3.3. Wind Loads and Bracing System

The distribution of wind loads on the tower is similar to those of a cylindrical body. In addition to a compressive stress on the front part of the structure the dominant influences are lateral suction forces. These suction forces pull apart the tower to the sides. The size of the recognized expense corresponds to the values for temporary structures in accordance with DIN 4112. To increase the radial stiffness and that way keeping the structure from global buckling, a spoke-wheel was added the structure. This kind of restraining mechanism has been described by Alpermann and Gengnagel for arcs [5] or membrane restrained columns and girders [4]. A set of tension cables is connected to the centre points of each storeys membrane surfaces. That way opposing suction forces can be short-circuited and the deflection in ground-plot can be reduced dramatically.



Figure 9: Wind load distribution and bracing system.

## 3.4. Prototype Analysis

## 3.4.1. Prototype 01 – Frame Only

The first analysed model is a tower with 6 floors and 6 support feet. The following tables summarize the results from the 14mm beam simulations. The maximum stress values, stress utilisation and maximum displacement. The stresses given here *do not include residual stresses* from form finding [FF]. Regarding the simulation, it is quite clear to see that the stiffer the model, the more likely to converge the FE simulation is. This explains some of the earlier challenges with the simulations: we are dealing with a very soft structure.

	2		Prestress [P]	Dead load [DL]	Wind [W]	1.35*DL + 1.5*W
	Max stress	[Mpa]	91.7	91.6	502.6	730
	Utilisation	[%]	36%	36%	196%	285%
	Max displacement	[mm]	1	9	438	1303
	Sofistik convergence		good	good	good	good
	Comments					
Basic model + membrane + extra legs	Unfactored deflection	2				

Figure 10: Stresses, utilisation and displacement of the basic model

#### *3.4.2. Prototype 02 – adding the Membrane*

After various iterations the tower was increased to 8 leg/8 storey subdivision. The ground floor contains 4 openings while every second side is closed with membrane for structural reasons. The convergence of the system was quite good and the simulation produced sensible structural output. The stress analysis of the tower follows a two-step process

- 1. Bending radii for each storey are taken from the particle-spring based form-found model and the initial stress level from bending the rods into shape is calculated.
- 2. Stresses of the FE-simulation are analysed and added to the stress level of the form-finding

Storey	r	FF-Model					FE-Analysis		Total	Total
	Ø [mm]	Radius	Stress [Mpa]	Utilisation [%]	Rese [Mpa]	rve [%]	Stress [Mpa]	Utilisation [%]	Stress [Mpa]	Utilisation [%]
8	8	670	149	60	101	40	63	25	212	85
7	8	580	172	69	78	31	103	41	275	110
6	8	590	169	68	81	32	107	43	276	110
5	8	600	167	67	83	33	78	31	245	98
4	8	620	161	65	89	35	88	35	249	100
3	10	680	184	74	66	26	56	22	240	96
2	12	720	208	83	42	17	66	26	274	110
1	12	990	152	61	98	39	56	22	208	83

Figure 11: Stresses of chosen design

The above mentioned results from the bi-axial testing from the *Essener Labor für leichte Flächentragwerke* came only in during the actual production of the knit. With the new, 50-times lower Young's modulus a converging calculation could not be achieved.

#### 3.5 Conclusion after FE-Analysis

Crucial to the stability of the chosen structure under wind load is a stabilisation of the curved compression elements via the fabric. For this, a certain stiffness of the fabric is required. The knitted membrane was so soft, that precisely this effect could not be sufficiently established. With the given

material properties, the FE simulation could only resist 20% of the actual load level. While the FE model is a precise in the simulation of the rods and membrane, its representation of the joints was also subject to imprecisions. In the final prototype the rods are only connected to one another by shared pockets in the fabric, whereas in the FE model, the rods are rigidly fixed together at the intersecting node. The effects of friction within the textile tubes and pockets were not simulated. Extensive testing would be required to establish a precise representation of node behaviour in FE.



Figure 12: Different intersections in different environments.

#### 4. Discussion and further work

The tower was built in Copenhagen and under first impressions performed quite well under external loads. The overall stiffness seemed to be higher than the simulation suggested. This is due to composite effects from friction, additional zip ties and geometrical stabilisation which were not fully accounted for in the models. Furthermore, overstressing of the GFRP rods does not result in their sudden failure but instead in subtle and non-catastrophic breaking of individual edge fibres.

The modelling pipeline performed very satisfactorily for generating and iterating design instances. While the bending radii analysis was instrumental to this, an increased integration of material properties in the form finding is desirable. This might include torsion and load cases as well as providing more modes for visualising analysis results. The current form finding does not model realistic properties of the membrane and the resulting interaction and deformation of the adjacent bending members is not taken into account. This interaction turned out to be crucial in the physical construction. A higher degree of integration between design modelling and structural analysis FEA modelling would also be desirable. First steps have been taken to further explore this.

The project demonstrates, that a complex hybrid structure of bending active rods and membrane systems can be form found and analysed using an interconnected particle spring and FE simulation. As particle spring does not simulate external and internal forces a subsequent iteration with FE allows to specify material and dimensions. The combination of the flexible particle-spring and the precise FE-world can lead to satisfactory design outputs as well as solid analysis. A higher degree of integration of the FE-analysis will reduce the feedback time and increase the degree of informational feedback to the design process. Related work [10] shows further potential in this combination of tools, as the integration of anisotropic behaviour in both tools would create a design environment, which allows for the design of interacting material and structural behaviour all between macro and meso scale. However current FE tools are not able to precisely simulate the highly nonlinear material behaviour of the knitted fabric. The FE- model is very sensitive on changes of stiffness and force

parameters and can hence not follow the fast design iterations, which are enabled by the particle spring system.

#### Acknowledgements

The project is funded by The Danish Council for Independent Research (DFF). Membranes were developed by Fibrenamics, Universidade do Minho, Guimarães, Portugal. Further development and fabrication took place with AFF a. ferreira & filhos, sa, Caldas de Vizela, Portugal. Mechanical testing of the knit was conducted by University Duisburg-Essen, Laboratory for Lightweight Structures. The Tower was exhibited at Designmuseum Danmark. We wish to thank: Daniel Piker for involving us in testing Joey/Kangaroo2. Ida Katrine Friis Tinning, Dongil Kim, Henrik Leander Evers, Esben Clausen Nørgaard and the students of CITAstudio for their tireless support.

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